



NATIONAL ADVISORY COMMITTEE  
FOR AERONAUTICS

TECHNICAL NOTE 3262

STARTING AND OPERATING LIMITS OF TWO SUPERSONIC WIND  
TUNNELS UTILIZING AUXILIARY AIR INJECTION

## DOWNSTREAM OF THE TEST SECTION

By Henry R. Hunczak and Morris D. Rousso

Lewis Flight Propulsion Laboratory  
Cleveland, Ohio



Washington .

September 1954

AF-111  
 TECHNICAL  
 1971



0066040

## NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

## TECHNICAL NOTE 3262

STARTING AND OPERATING LIMITS OF TWO SUPERSONIC WIND  
TUNNELS UTILIZING AUXILIARY AIR INJECTION  
DOWNSTREAM OF THE TEST SECTION

By Henry R. Hunczak and Morris D. Rousso

## SUMMARY

The starting and operating pressure ratios were determined for two supersonic wind tunnels which employed air injectors to supplement the primary pumping systems of the tunnels. Data are presented for tunnels operating at Mach numbers 3.85, 3.05, and 2.87 over a range of injector-to-tunnel mass-flow ratios of 0.5 to 1.35. At Mach number 3.85, the starting pressure ratio of 9.8 without injectors but with a fixed second throat was reduced to 4.68 with injectors operating at an injector-to-tunnel mass-flow ratio of 1.27. The running pressure ratio was lowered from 8.3 to 4.5. Corresponding reductions at Mach number 3.05 were from 4.5 to 2.71 for starting and from 4.5 to 2.37 for running at a mass-flow ratio of 0.9. Those at Mach number 2.87 were from 3.8 to 2.43 for starting and from 3.8 to 2.13 for running at a mass-flow ratio of 1.35. The data indicate that the tunnels with injectors operated at pressure ratios approximately 20 percent greater than the theoretically predicted values.

## INTRODUCTION

The use of auxiliary air injection downstream of a supersonic wind tunnel test section was initiated at the NACA Ames laboratory, and the preliminary results showed that this method of tunnel operation reduced the pressure ratio required to start and run a supersonic tunnel. However, the injection of mass flow requires a more complex ducting system and generally results in added power requirements. The development of the specific theoretical analysis and results of the first experimental work are presented in reference 1. The method of operation is similar to that of induction tunnels analyzed in reference 2.

3165

CE-1

The pressure-ratio reduction attainable with auxiliary injection suggests its use as a means of extending the operational limits of supersonic tunnels that are restricted by the pressure-ratio capacity of their pumping facilities. Two of several such applications as suggested in reference 1 are that of extending the Mach number range of continuous-flow supersonic tunnels and the running time of blow-down tunnels.

The feasibility of such proposals, however, depends on the effectiveness of the injectors in reducing the tunnel pressure ratio without excessive increases in the total weight flow. Because of the limitations due to necessary assumptions in injector theory at the present time, injector performance can be accurately determined only by experiment. The work at Ames constitutes the only known experiments wherein the injectors are supplementary to the primary pumping system. Pilot studies to obtain additional information were conducted in the Lewis 2- by 2-foot supersonic tunnel, which was operated at Mach number 3.85, and in the 18- by 18-inch tunnel, which was operated at Mach numbers of 3.05 and 2.87. The results are presented herein.

#### SYMBOLS

The following symbols are used in this report:

A	area, sq ft
$c_p$	specific heat at constant pressure, Btu/(lb)(°F)
$c_v$	specific heat at constant volume, Btu/(lb)(°F)
D	Mach number function, $M \left( 1 + \frac{\gamma-1}{2} M^2 \right)^{-\frac{\gamma+1}{2(\gamma-1)}}$
G	Mach number function, $(1 + \gamma M^2) \left( 1 + \frac{\gamma-1}{2} M^2 \right)^{-\frac{\gamma}{\gamma-1}}$
g	acceleration of gravity, 32.2 ft/sec <sup>2</sup>
M	Mach number
m	mass flow, slugs/sec
N	Mach number function, D/G
P	stagnation pressure, lb/sq ft absolute

- p static pressure, lb/sq ft absolute  
R gas constant, 53.3 (ft-lb)/(lb-°F)  
r ratio of injector to tunnel mass flow  
T stagnation temperature, °R  
γ ratio of specific heats, 1.400  
θ ratio of injector to tunnel stagnation temperature

## Subscripts:

- 0 intake of tunnel nozzle  
1 tunnel test section  
2 tunnel exit  
3 end of mixing zone  
4 minimum area of tunnel diffuser  
a atmospheric conditions  
d end of tunnel diffuser  
j injector  
t tunnel discharge tank

## Superscript:

- \* sonic conditions

## THEORY OF OPERATION

To start and to operate a supersonic tunnel require pressure ratios across the tunnel circuit high enough to overcome the viscous and terminal shock losses of the stream flow. These losses increase with free-stream Mach number and must be overcome by the tunnel pumping facilities. Auxiliary air injection can reduce the pressure-ratio requirements of the drive system if the capacity of the facilities is sufficiently large to handle the increased flow associated with injection. The reduction in pressure ratio is analogous to that obtained by placing an auxiliary pump in the tunnel circuit between the end-point measuring stations. The mechanics of the flow process are as

follows: In starting, the main tunnel flow leaves the test section at a reduced total pressure and velocity due to shock losses. If auxiliary air is injected at this point with high total pressure and velocity, mixing of the two streams results in a combined flow having intermediate losses. For the tunnel running condition, the terminal shock losses occur downstream of the auxiliary air injection point and the two flows may or may not mix prior to this shock. In either case, the injected air decelerates the tunnel flow upstream of the terminal shocks, thus reducing the shock losses.

A simplified one-dimensional analysis of injector performance was made by applying the equations of energy, momentum, and continuity. The factors taken into consideration are tunnel free-stream Mach number, injector Mach number, contraction of the tunnel stream flow before injection (upstream contraction), contraction of the stream flow after injection (downstream contraction), type of tunnel-injector flow mixing process (constant-pressure or constant-area mixing), and ratios of injector to tunnel mass flow, stagnation pressure, and stagnation temperature. For convenience, the equations and their use are outlined in the appendix.

The theoretical analysis of reference 1 indicates that the constant-area mixing process is superior to the constant-pressure mixing; and that if both tunnel and injector are operated from a single air-flow supply, a ratio of stagnation pressures of unity is desirable. The desirability of upstream and downstream contraction was also indicated; however, in the present investigation upstream contraction was not used because data for a subsequent design employing downstream contraction alone were desired.

## APPARATUS AND PROCEDURE

### Tunnel Installation

The 2- by 2-foot and 18- by 18-inch supersonic tunnels (fig. 1) are of the single-pass, continuous-air-flow type and are connected in parallel between air drying and exhausting facilities as shown schematically in figure 1(a). The air flow is drawn through the tunnel by means of piston-type exhaustors with the selection of the tunnel to be operated being made by means of the 48-inch gate valves. The tunnel stagnation pressure is close to atmospheric. The back pressure on tunnel diffusers is regulated by bleeding air from the atmosphere to the tunnel discharge tank through the 24-inch gate valve. At all times the injectors drew air directly from the atmosphere without drying or throttling.

3165

2- by 2-foot supersonic wind tunnel and injector configuration. - The 2- by 2-foot supersonic wind tunnel and injector are shown in figure 2. The tunnel nozzle contour plates form the top and bottom walls, and their area ratio uncorrected for boundary layer corresponds to a theoretical Mach number of 4.00. Tunnel calibration with a wedge has indicated the Mach number of the flow to be 3.85 through the test section. The tunnel injector incorporates a flexible throat section, movable wall segments, a short constant-area mixing length, and downstream contraction to an area equal to that of the tunnel test section. This contraction is not only theoretically desirable but was necessary to avoid extensive alteration to the tunnel subsonic diffuser.

The variations in injector configuration included changing the injector throat area, offset, and injection angle. The ranges of variable and test conditions were as follows:

- (a) Ratio of injector to tunnel mass flow, 0.600 to 1.25 (based on one-dimensional sonic flow area and measured stagnation temperatures and pressures)
- (b)  $5^{\circ}$  injection angle with 4, 2, and  $1\frac{1}{16}$ -inch offsets
- (c) 2-inch offset with  $5^{\circ}$ ,  $10^{\circ}$ ,  $15^{\circ}$ , and  $20^{\circ}$  injection angles
- (d) Tunnel stagnation temperature,  $200^{\circ}$  F; injector stagnation temperature, approximately  $50^{\circ}$  to  $90^{\circ}$  F
- (e) Tunnel and injector stagnation pressures, approximately atmospheric
- (f) Tunnel air dew point,  $-5^{\circ}$  to  $-15^{\circ}$  F

The injector exit Mach number varied with injector-to-tunnel mass-flow ratio and injector offset as shown in figure 3. Because the injector walls were not parallel to the tunnel walls at the exit for the  $10^{\circ}$ ,  $15^{\circ}$ , and  $20^{\circ}$  offsets, the Mach number as calculated from the throat-to-exit area ratio varied also with injection angle, as may be seen by comparing the  $5^{\circ}$  and  $15^{\circ}$  curves for a 2-inch offset.

18- by 18-inch wind tunnel and injector configurations. - The 18- by 18-inch wind tunnel and injector configurations are shown in figure 4. The unmodified tunnel nozzle theoretically expands the flow to a Mach number of 3.20 when uncorrected for boundary layer. Calibration with a wedge indicates a test-section Mach number of 3.05.

To extend the Mach number range of the investigation, the nozzle and the test section were modified by reducing the width of the tunnel with two  $1\frac{1}{2}$ -inch inserts. Two struts spanned the tunnel as support

columns to prevent a collapse of the upstream edge of the inserts (see detail A in fig. 4). The theoretical Mach number based on the geometry was 3.01, and calibration with a pitot-static probe indicated a Mach number of 2.87 at the test-section exit.

Two injector configurations were investigated. Their design was governed by space and structural limitations that prohibited the incorporation of any constant-area mixing length. A downstream contraction to the test-section area of the unmodified tunnel was again mandatory and the injector geometries did not conform to either a constant-area or a constant-pressure mixing process.

The first configuration consisted of only the fixed contoured wooden blocks shown in figure 4, which had a  $15^\circ$  injection angle with respect to the tunnel axis. The mass-flow ratio of this configuration was varied by reducing the 18-inch injector height with inserts contoured to fit the flow passage, thus permitting only partial injection. The second injector configuration employed a flexible wall using the outer wooden blocks of the first configuration as structural members only (see also fig. 5). Stresses on the flexible wall were reduced by venting the space behind it to the tunnel transition section. Independent control of both the injector throat and the exit area (mass flow and Mach number) was obtained by either flexing the wall or rotating the entire assembly or both.

The first injector configuration was investigated only at a tunnel Mach number of 3.05 over a mass-flow-ratio range from 0.825 to 0.900 at an injector Mach number of 2.07. Although the mass-ratio range with inserts was limited, a variety of insert arrangements which injected equal mass flows over only a part of each tunnel side were investigated.

Most of the data presented were obtained with the second injector configuration which employed flexible plates. The ranges of injected flow variables are as follows:

At Mach number 3.05:

- (a) Ratio of injector to tunnel mass flow, 0.5 to 1.0
- (b) Injector Mach numbers, 1.40 to 2.20

At Mach number 2.87:

- (a) Ratio of injector to tunnel mass flow, 0.9 to 1.40
- (b) Injector Mach numbers, 1.70 to 2.20

The test conditions were as follows:

- (a) Tunnel and injector stagnation temperature,  $50^{\circ}$  to  $90^{\circ}$  F
- (b) Tunnel and injector stagnation pressure, approximately atmospheric
- (c) Tunnel air-flow dew point,  $-7^{\circ}$  to  $-33^{\circ}$  F.

#### Instrumentation and Data Reduction

The instrumentation (see also fig. 1) consisted of the following items: (a) four pitot tubes and four thermocouples located upstream of the tunnel nozzle (station 0); (b) static orifices along the center line of the tunnel nozzle, test section, injector, and downstream contraction side plates; (c) a five-tube pitot rake at the end of the tunnel diffuser (station d); (d) a static orifice in the tunnel discharge tank (station T); and (e) a thermometer at the injector intake.

The stagnation pressures were based on area weighted averages. The tunnel and injector mass flows were calculated using the average stagnation pressures, stagnation temperatures, and sonic throat areas, assuming one-dimensional isentropic flow from the measuring stations to the throat stations.

#### Methods of Tunnel Starting

Starting of the 2- by 2-foot tunnel was possible by two different procedures. In the conventional procedure, both 48-inch gate valves as well as the 24-inch bleed valve were opened and the exhausters started. The pressure ratio across the tunnel and injectors was then controlled by throttling the bleed air. In this manner the starting pressure ratio could be approached gradually. The maximum pressure ratio attainable with the pumping facilities when handling both tunnel and injector flow (no bleed air) is presented in figure 6 as a function of injector-to-tunnel mass-flow ratio.

In the low-density start, the upstream gate valve and bleed valve were closed when the exhausters were started. The flow through the injectors was thus established first, evacuating the tunnel and piping to the upstream valve. The upstream valve was then opened for the start.

A low-density start aided the tunnel starting in two ways. First, a pressure ratio higher than that attainable for the operating condition was temporarily available. This pressure ratio is shown as a curve of



$P_a/P_t$  in figure 6 and is a result of the reduced flow to the exhausters when the upstream valve was closed. The aspirating action of the injectors further increased the pressure ratio between the atmospheric side of the upstream valve and the tunnel test section as shown by the curve of  $P_a/P_1$ . Both pressure ratios are plotted in figure 6 against values of injector-to-tunnel mass-flow ratio for the tunnel running condition. The second advantage resulted from the fact that the starting process occurred at a reduced stagnation pressure and, hence, reduced tunnel mass rate. The injector-to-tunnel mass-flow ratio during starting was therefore increased which, as will be shown subsequently, reduced the starting requirements.

For starting, the evacuated piping acted as a vacuum storage tank with the actual pressure ratio across the nozzle depending on the volume of the piping and the rate at which the upstream valve was opened. The supersonic flow in the test section was established at a pressure ratio between that indicated by the curves of  $P_a/P_t$  for the low-density and the conventional starts. In addition to aiding the tunnel starting, the low-density start reduced the loads on the model in the test section and the duration of the starting process.

## RESULTS AND DISCUSSION

The theoretical performance of both tunnel injector configurations is presented in figures 7 and 8. The performance calculated for Mach number 4.0 corresponds to the geometry of the 2- by 2-foot supersonic tunnel and test conditions  $P_j/P_1$  of 1.0 and  $T_j/T_0$  of 0.788. For this configuration, the injector exit Mach number varies with mass ratio as previously indicated in figure 3. The data at Mach numbers 3.20 and 3.01 (figs. 8(a) and (b)) correspond to the 18- by 18-inch tunnel and the injector configuration which employed the flexible plate. For these two configurations  $P_j/P_1$  and  $T_j/T_1$  were both 1.0 and the injector exit Mach numbers remained fixed at 2.10 and 1.80, respectively.

All calculations are based on one-dimensional flow theory and a constant-area mixing process. Subsonic diffuser or wall friction losses are not included and the geometric tunnel and injector areas are uncorrected for boundary layer.

In addition to the starting and running pressure ratios, theoretical choking limits are indicated. These limits represent the maximum mass injection, which, if exceeded, chokes the minimum downstream area before the tunnel starts. The solid curves of figures 7 and 8 from which one limit is obtained are based on the assumption of complete flow mixing. The second limit is obtained for the assumption of no flow mixing. These two limits should represent the extremes of the actual flow-choking process.

3165  
CE-2

The theoretical analysis indicates that at Mach number 4.0 (fig. 7) the tunnel starting should not be limited by choking in the injected mass range investigated. At Mach number 3.20 (fig. 8(a)), the choking mass ratio of 0.93 for complete mixing is close to that of 0.96 for no flow mixing. A reduction in test-section Mach number to 3.01 (fig. 8(b)) lowers the starting pressure ratio and increases the permissible mass-ratio injection. Again the choking limits for complete flow mixing ( $m_j/m_0 = 1.52$ ) and no flow mixing ( $m_j/m_0 = 1.42$ ) are close.

Relatively unaffected by the change in Mach number from 3.20 to 3.01 were the running pressure ratios and choking pressure ratios for complete flow mixing at mass ratios greater than 0.60. Above this injected mass ratio, the downstream contraction is the minimum area, and the choking pressure ratios for a given mass injector are identical. The running pressure ratios differ only because of a slight change in the supersonic mixing losses. Below a mass ratio of 0.6 no downstream contraction exists at Mach number 3.01. For the combined flow, the tunnel plus injector exit area is the minimum. Therefore, the choking pressure ratio is less than at Mach number 3.20, and the running and starting pressure ratios at Mach number 3.01 coincide.

2- by 2-foot tunnel-injector performance. - The performance of the 2- by 2-foot tunnel operating with the various injector configurations is presented in figure 9. The lowest running pressure ratios are indicated for the 2-inch offset with a  $5^\circ$  injection angle and no model in the test section. However, a low-density start was required to establish the test-section flow and insertion of a split-wing ram-jet model to simulate pressure losses prevented continuous operation.

Running pressure ratios with a model to simulate losses were obtained with a 2-inch offset and either a  $10^\circ$  or  $15^\circ$  injection angle. The test-section flow could be established with a conventional start. The experimental curves are drawn through the data for these points, and a comparison with the theoretical curves (from fig. 7) indicates a fair agreement between their slopes. The tunnel which employed a fixed second throat prior to the installation of the injector required pressure ratios of approximately 9.8 to start and 8.3 to run. Operation with injectors at a mass ratio of 1.27 lowered the starting and running pressure-ratio requirements to 4.68 and 4.5, respectively. The  $5^\circ$  injector with a 4-inch offset and the  $20^\circ$  injector with a 2-inch offset had running pressure ratios greater than that available from the pumping facilities, although the test-section flow could be established momentarily with a low-density start.

18- by 18-inch tunnel injector performance. - The 18- by 18-inch tunnel-injector performance is presented in figure 10. The initial injector with inserts (partial injection) permitted starting up to a mass ratio of 0.85 (fig. 10(a)). Starting and running pressure ratios were identical and were not affected by variations in the location of the injector passages.

For the flexible wall configuration, choking at the downstream contraction prevented starting at mass ratios above 0.90, a value close to that theoretically predicted. Data taken for a systematic variation of injector Mach number from 1.40 to 2.30 established the optimum value at 2.10 for both starting and running. This value was critical because for values below 1.80 or above 2.20 choking prevented starting. At a mass ratio of 0.9, the starting pressure ratio was 2.71, and that for running was 2.57. Without injectors the tunnel required a pressure ratio of 4.5 for both running and starting.

At Mach number 2.87 (fig. 10(b)) the injector-tunnel mass ratio was limited to a minimum value of 0.90 by stresses in the flexible wall, and extended to a maximum of 1.325 prior to choking on the start. The agreement with the theoretically predicted choking limit was not so good as at Mach number 3.05. This increased departure from theory may be a result of losses from disturbances generated by the support struts at the beginning of the test section. Again the injector Mach number was critical in the starting process, the optimum value being approximately 1.80. The minimum pressure ratio for starting, 2.43, and that for running, 2.13, were obtained at a mass ratio of 1.35. With no injection flow, both pressure ratios were 3.8.

During the course of the investigation, two values of the starting pressure ratio existed in many instances. The data of figure 10 are the higher of the two values and were obtained when the flow separated from one wall of the tunnel test section during the starting process. With no flow separation, the starting pressure ratios were almost the same as the running values. However, once the tunnel was started, a reduction in pressure ratio below the running value invariably resulted in a separated flow. This separation persisted with supersonic flow partially filling the test section until the pressure ratio was increased to the higher starting value. Alternatively, if the pressure ratio was reduced below approximately 1.90, the separation was eliminated and a subsonic flow filled the test section.

Comparison of experimental with theoretical performance. - A comparison of the experimental with the theoretical performance as calculated from the data of figures 7 to 10 is presented in figure 11. The theoretical performance as previously discussed does not include wall friction or subsonic diffuser losses.

At all three Mach numbers, the experimental values of the running pressure ratio were found to be approximately 20 percent greater than the theoretical. At Mach numbers 3.85 and 3.05, the experimental starting values are within 10 percent of the theoretical and are believed to be conservative because they represent the higher of the two observed values. For the case of attached flow to the tunnel wall throughout the starting process, the lower value of starting pressure ratio in some

instances is less than theoretical (comparison not shown in fig. 11). In this connection, viscous corrections would lower the theoretical values. At Mach number 2.87, the agreement was not so good as for Mach number 3.85 or 3.05. For this case it was felt that two sources of total-pressure loss may account for the increased difference. First, a loss is incurred from the strong disturbances generated by the test-section-insert support struts and the discontinuity at the juncture of the test-section insert with the nozzle contour as previously indicated. Second, the viscous losses are increased because the test section with the inserts is effectively longer than would normally be employed at that Mach number.

### SUMMARY OF RESULTS

An investigation conducted to determine the starting and running performance of two supersonic wind tunnels operating with the aid of auxiliary air injection downstream of the test section gave the following results:

1. Starting and running pressure ratios were appreciably reduced as listed in the following table. Those at Mach number 3.85 without injectors were obtained with a fixed second throat.

Tunnel Mach number	With injection			Without injection	
	Injector mass-flow ratio	Starting pressure ratio	Running pressure ratio	Starting pressure ratio	Running pressure ratio
3.85	1.27	4.68	4.50	9.8	8.3
3.05	.90	2.71	2.57	4.5	4.5
2.87	1.35	2.43	2.13	3.8	3.8

2. The experimental running pressure ratios which include subsonic diffuser losses were approximately 20 percent greater than the theoretically predicted ratios which did not include any friction or diffuser losses and which assumed ideal nozzle flow. The starting values were within 10 percent of theoretical.

3. The optimum angle of injection with respect to the tunnel axis was  $10^{\circ}$  to  $15^{\circ}$  at Mach number 3.85. These values were critical in this installation because lower or higher injection angles required running pressure ratios, with a model in the tunnel section, greater than those available from the pumping equipment.

3165

CE-2 back

4. At tunnel Mach numbers of 3.05 and 2.87, the optimum injector Mach numbers when using a  $15^\circ$  injection angle were 2.10 and 1.80, respectively. At injector Mach numbers above 2.20 or below 1.80 choking at the downstream contraction prevented starting.

Lewis Flight Propulsion Laboratory  
National Advisory Committee for Aeronautics  
Cleveland, Ohio, July 6, 1954

## APPENDIX - OUTLINE OF THEORETICAL ANALYSIS

The equations used to calculate the performance of the tunnel-injector configurations are presented and their application is outlined. The station locations as used are indicated in figure 1(b).

## Assumptions

- (1) One-dimensional adiabatic flow through tunnel, injector, and diffuser
- (2) No friction losses
- (3) A constant-area mixing process for the tunnel flow with injector flow, with all mixing complete at station 3
- (4)  $C_p$ ,  $C_v$ ,  $\gamma$ , and  $R$  constant

## Equations

The general equations of continuity, momentum, and energy for the constant-area flow mixing process are given in references 2 and 3. Between stations 3 and 4 they may be written in terms of stagnation pressure and temperature as

Continuity:

$$m_2 + m_j = m_3$$

or

$$\frac{P_1 A_1 D_1}{\sqrt{T_1}} = \frac{P_2 A_2 D_2}{\sqrt{T_2}} = \frac{1}{r} \frac{P_j A_j D_j}{\sqrt{T_j}} = \frac{1}{(1+r)} \frac{P_3 A_3 D_3}{\sqrt{T_3}} \quad (A1)$$

where

$$r = \frac{m_j}{m_2}$$

$$m = \sqrt{\frac{\gamma}{gR}} \frac{PAD}{\sqrt{T}}$$

and

$$D = M \left( 1 + \frac{\gamma-1}{2} M^2 \right)^{-\frac{\gamma+1}{2(\gamma-1)}}$$

Momentum:

$$P_2 A_2 G_2 + P_j A_j G_j = P_3 A_3 G_3 \quad (A2)$$

when

$$A_2 + A_j = A_3.$$

and where

$$G = \left(1 + \gamma M^2\right) \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{\gamma}{\gamma-1}}$$

Energy:

$$m_2 c_p T_2 + m_j c_p T_j = m_3 c_p T_3 \quad (A3)$$

which reduces to

$$\frac{T_3}{T_2} = \frac{1 + r\theta}{1 + r} = \frac{T_3}{T_1}$$

where

$$\theta = \frac{T_j}{T_2}$$

Combining equations (A1), (A2), and (A3) yields

$$N_3 = \frac{(1+r)^{1/2} (1+r\theta)^{1/2}}{\frac{1}{N_2} + \frac{r\theta^{1/2}}{N_j}} \quad (A4)$$

where

$$N = D/G = M \left(1 + \frac{\gamma-1}{2} M^2\right)^{1/2} \left(1 + \gamma M^2\right)^{-1}$$

A tabulation of  $D$ ,  $G$ , and  $N$  as well as several other useful functions of Mach number may be found in reference 3 for a range of Mach numbers from 0 to 5 in increments of 0.001.

The continuity and energy relations between the various stations yield the following equations: With upstream contraction,

$$D_2 = \frac{P_1 A_1 D_1}{P_2 A_2} \quad (A5)$$

Between stations 1 and 3,

$$\frac{P_1}{P_3} = \frac{1}{(1+r)^{1/2}} \frac{1}{(1+r\theta)^{1/2}} \left( \frac{A_2}{A_1} + r\theta^{1/2} \frac{P_1 D_1}{P_j D_j} \right) \frac{D_3}{D_1} \quad (A6)$$

With downstream contraction,

$$D_4 = \frac{A_3}{A_4} D_3 \quad (A7)$$

and when  $A_4 = A_2$ ,

$$D_4 = \left( \frac{A_2}{A_1} + r\theta^{1/2} \frac{P_1 D_1}{P_j D_j} \right) \frac{A_1}{A_2} D_3 \quad (A8)$$

The equations for the choking limits of either pressure ratio, area ratio, or mass ratio between station 1 and any other downstream station may be based on either the assumption of complete mixing or no mixing of the tunnel with injector flow.

For complete mixing, the choking pressure ratio between stations 1 and 4 is obtained from continuity and energy relations as

$$\left( \frac{P_1}{P_4} \right)^* = \frac{1}{(1+r)^{1/2}} \frac{1}{(1+r\theta)^{1/2}} \frac{A_4 D^*}{A_1 D_1} \quad (A9)$$

For a given Mach number at 1, should the pressure ratio and mass ratio be predetermined, the choking area ratio may be calculated. Thus, between stations 1 and 2,

$$\left( \frac{A_1}{A_2} \right)_{\max} = \frac{P_2 D^*}{P_1 D_1} \quad (A10)$$

for the starting condition of shock in the test section.



With no mixing of tunnel with injector air, a condition which might occur immediately downstream of stations 2 and j, the tunnel and injector air are assumed to attain a static-pressure equilibrium. Should  $p_j$  be greater than  $p_2$ , the injector air may expand and choke off the tunnel flow for the starting condition of shock in the test section and supersonic injector flow. For this condition,

$$\frac{A_3}{A_1} = \frac{A_2}{A_1} + \frac{A_j}{A_1} = \frac{A_{2,3}}{A_1} + \frac{A_{j,3}}{A_1}$$

where  $A_{2,3}$  and  $A_{j,3}$  are the areas occupied by the tunnel air and injection air, respectively, on attaining static-pressure equilibrium. Solving for  $A_1/A_2$  and substituting the continuity and energy relation give

$$\left(\frac{A_1}{A_2}\right)_{\max} = \frac{\frac{P_{2,3} D_{2,3}}{P_1 D_1}}{1 + r\theta^{1/2} D_{2,3} \frac{P_{2,3}}{P_1} \frac{P_1}{P_{j,3}} \left(\frac{1}{D_{j,3}} - \frac{1}{D_j}\right)} \quad (A11)$$

where  $\frac{P_{2,3}}{P_1}$  is the normal shock pressure recovery at the test section Mach number,  $D_{2,3} = D^* = 0.5787$ ,  $P_{j,3} = P_j$ , and  $D_{j,3}$  is found from  $P_{j,3} = P_{2,3}$ ; or

$$\frac{P_{j,3}}{P_{j,3}} = \frac{P_{2,3}}{P_{2,3}} \frac{P_{2,3}}{P_1} \frac{P_1}{P_j} = 0.528 \frac{P_{2,3}}{P_1} \frac{P_1}{P_j} \quad (A12)$$

The maximum upstream contraction as determined by equation (A11) (no mixing) is less than that determined by equation (A10) (complete mixing) when the tunnel static pressure  $p_2$  is less than the injector static pressure  $p_j$ . For this condition,  $M_j$  is less than  $M_{j,3}$  and  $\frac{1}{D_{j,3}} - \frac{1}{D_j}$  is greater than zero. Should  $p_2$  be greater than  $p_j$ ,

$\frac{1}{D_{j,3}} - \frac{1}{D_j}$  is less than zero, equation (A11) is no longer valid, and equation (A10) determines the upstream contraction limit.

The parameters  $D$ ,  $G$ , and  $N$  as well as the normal shock pressure recovery and local static-to-total pressure ratio are all functions of  $M$ . Evaluation of any one function enables the others to be found from

the tables such as those of reference 3. However, two precautions must be considered. First, although the parameter  $N$  is constant through a normal shock (at constant area), each value of  $D$ ,  $G$ , and  $N$  has a subsonic and supersonic solution of  $M$  and the correct solution corresponding to the physics of the flow must be used. Second, the choking limitations such as those given by equations (A9) to (A11) should not be exceeded, and, in fact, the maximum permissible area contractions determined by them should be reduced because of viscous effects.

### Theoretical Analysis Procedure

Equation (A4) is used to find the resultant Mach number  $M_3$  after the mixing process. For the starting condition, a normal shock is assumed in the tunnel test section, station 1;  $N_2$  is found through the use of equation (A5) where  $D_2$  and  $M_2$  are subsonic; if upstream contraction is used, it has a limiting value as determined by either equation (A10) or (A11); and  $M_3$  must be the subsonic value corresponding to  $N_3$ . Equation (A6) is then solved for the starting pressure ratio.

For the running condition, the normal shock which terminates the supersonic flow is assumed to be at station 4.  $P_1 = P_2$  in equation (A5), and  $D_2$ ,  $M_2$ , and  $N_2$  are supersonic solutions. The use of a supersonic solution for  $D_3$  in equation (A6) gives the supersonic mixing pressure ratio. Then for the supersonic flow ahead of the shock at station 4,  $P_4 = P_3$  and equations (A7) or (A8) are used to determine  $D_4$  and the normal shock recovery at  $M_4$ . Dividing the supersonic mixing pressure ratio by the normal shock recovery at  $M_4$  gives the running pressure ratio.

Should a subsonic solution for  $D_3$  be used in equation (A6) for the running condition, the pressure ratio  $P_1/P_3$  will correspond to a flow condition where a normal shock is positioned at station 3. This solution corresponds to the running pressure ratio for a geometry with no downstream contraction. With no upstream contraction ( $A_1 = A_2$ ), the starting normal shock may be assumed to occur anywhere between stations 1 and 3 because  $N_1 = N_2$  and  $P_1 D_1 = P_2 D_2$  across the shock; a subsonic solution for  $D_3$  in equation (A6) gives the starting pressure ratio.

### REFERENCES

1. Spiegel, Joseph M., Hofstetter, Robert U., and Kuehn, Donald M.: Applications of Auxiliary Air Ejectors to Supersonic Wind Tunnels. NACA RM A53I01, 1953.
2. Crocco, Luigi: High Speed Wind Tunnels. Trans. No. 366, Air Materiel Command, AAF, July 30, 1943.

5165

CE-3

3. Foa, J. V.: Mach Number Functions for Ideal Diatomic Gases. Cornell Aero. Lab., Inc., Oct. 1949.
4. Weatherston, Roger: Mixing of Any Number of Streams in a Duct of Constant Cross-Sectional Area. Jour. Aero. Sci., vol. 16, no. 11, Nov. 1949, pp. 697-698; 704.

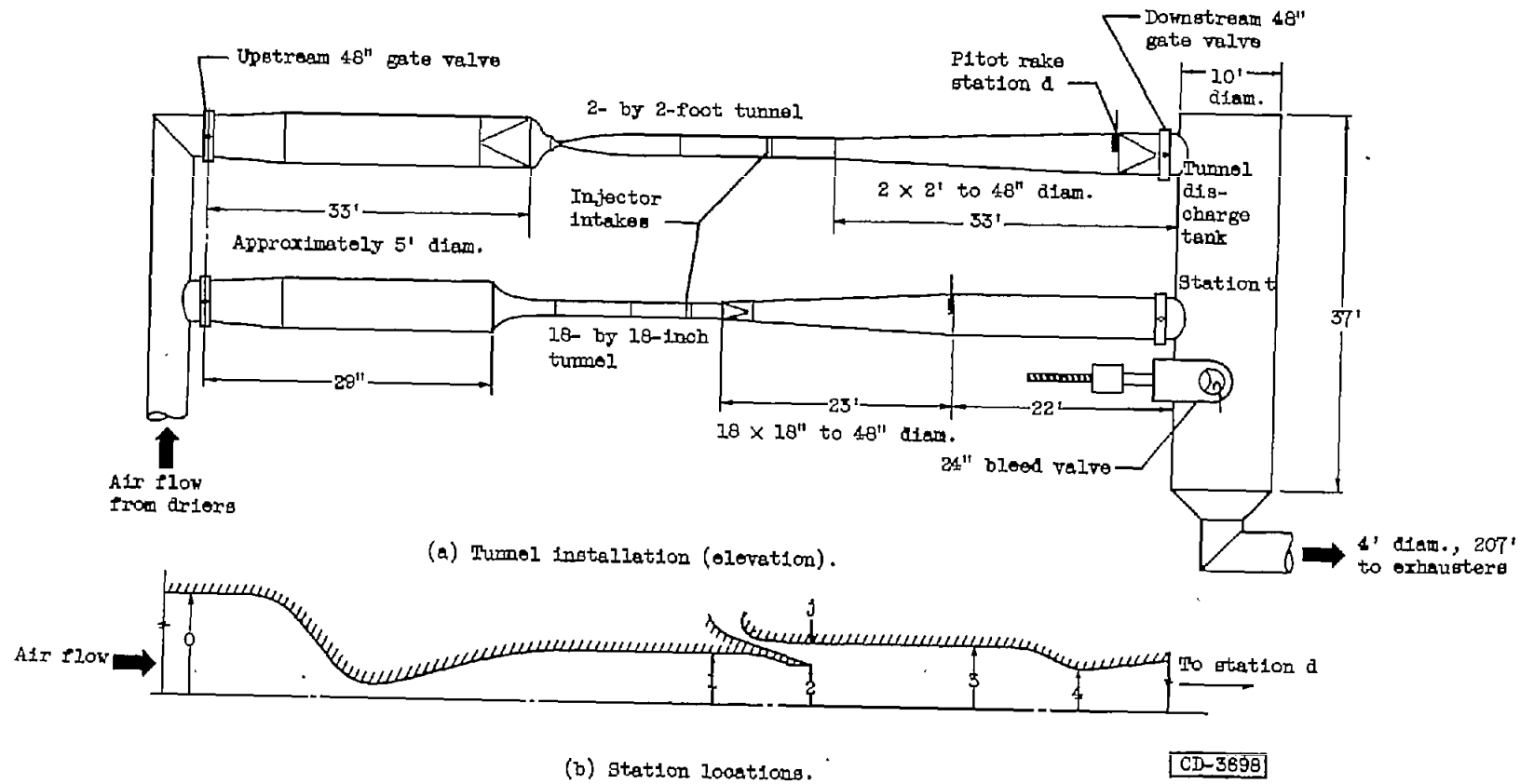


Figure 1. - Sketch of tunnel installation and station locations.

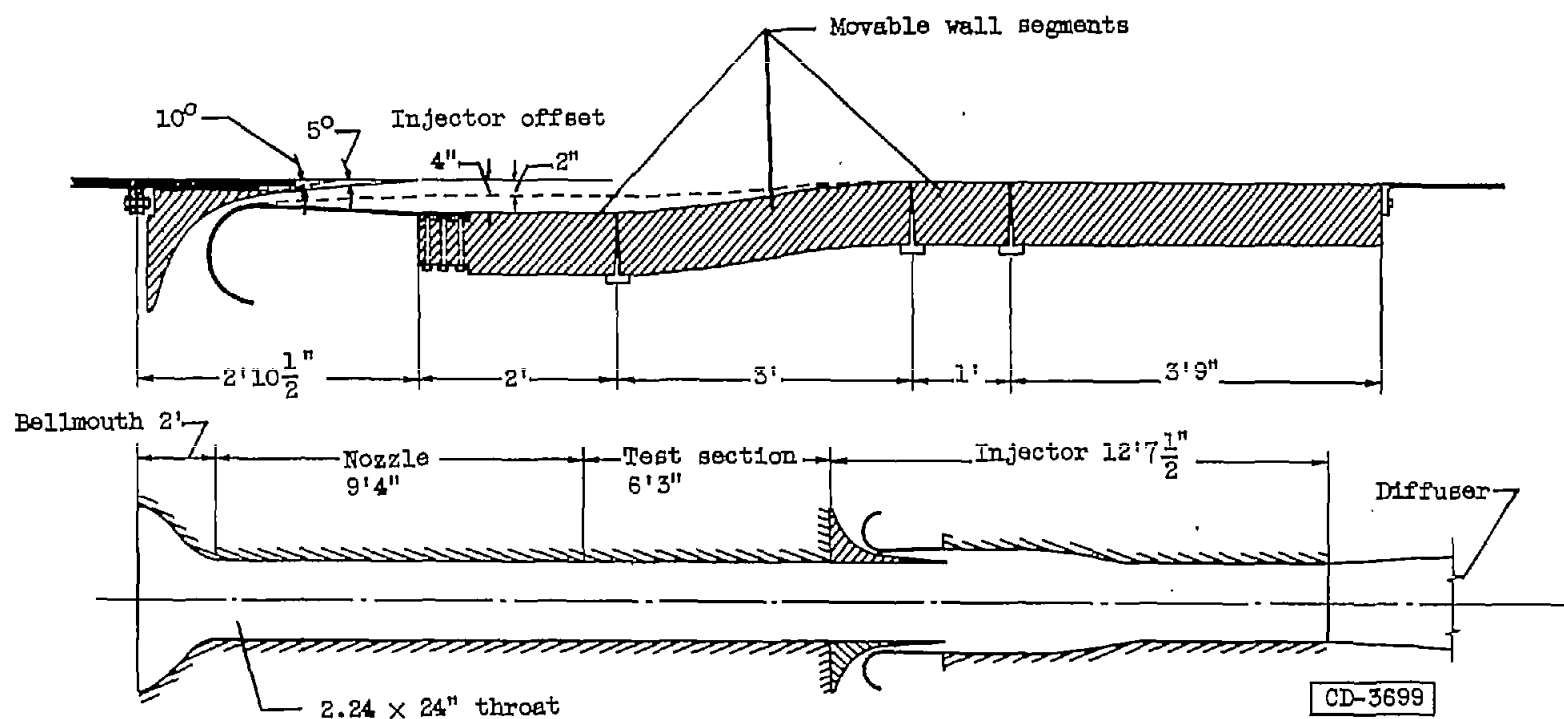


Figure 2. - Top view of injector installation for 2- by 2-foot supersonic wind tunnel (plan view).

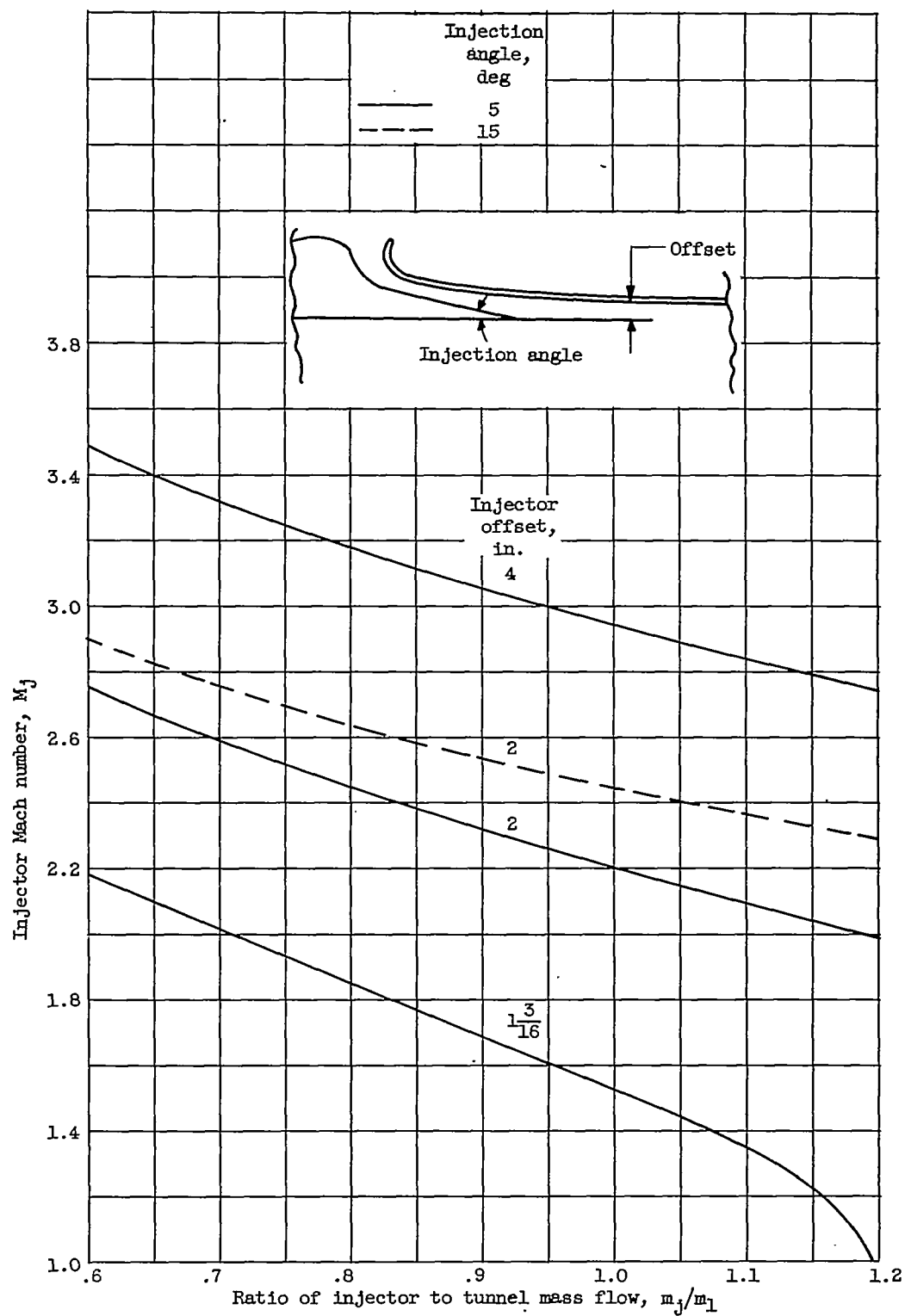


Figure 3. - Theoretical variation of injector Mach number with injector offset and injection angle. 2- by 2-foot supersonic tunnel; Mach number, 3.85.

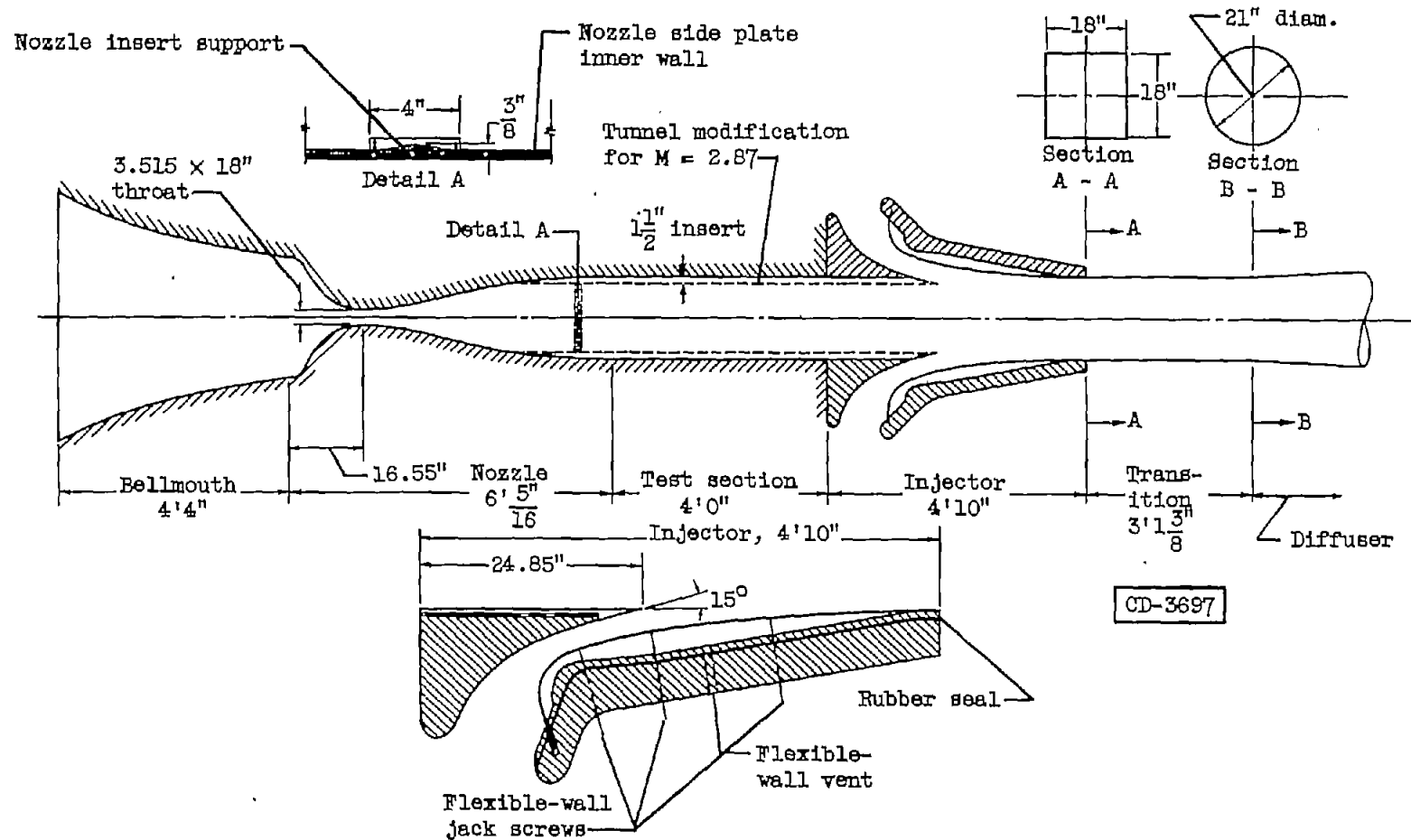


Figure 4. - Injector installation for 18- by 18-inch supersonic tunnel (plan view).



Figure 5. - Photograph of 18- by 18-inch tunnel injector.



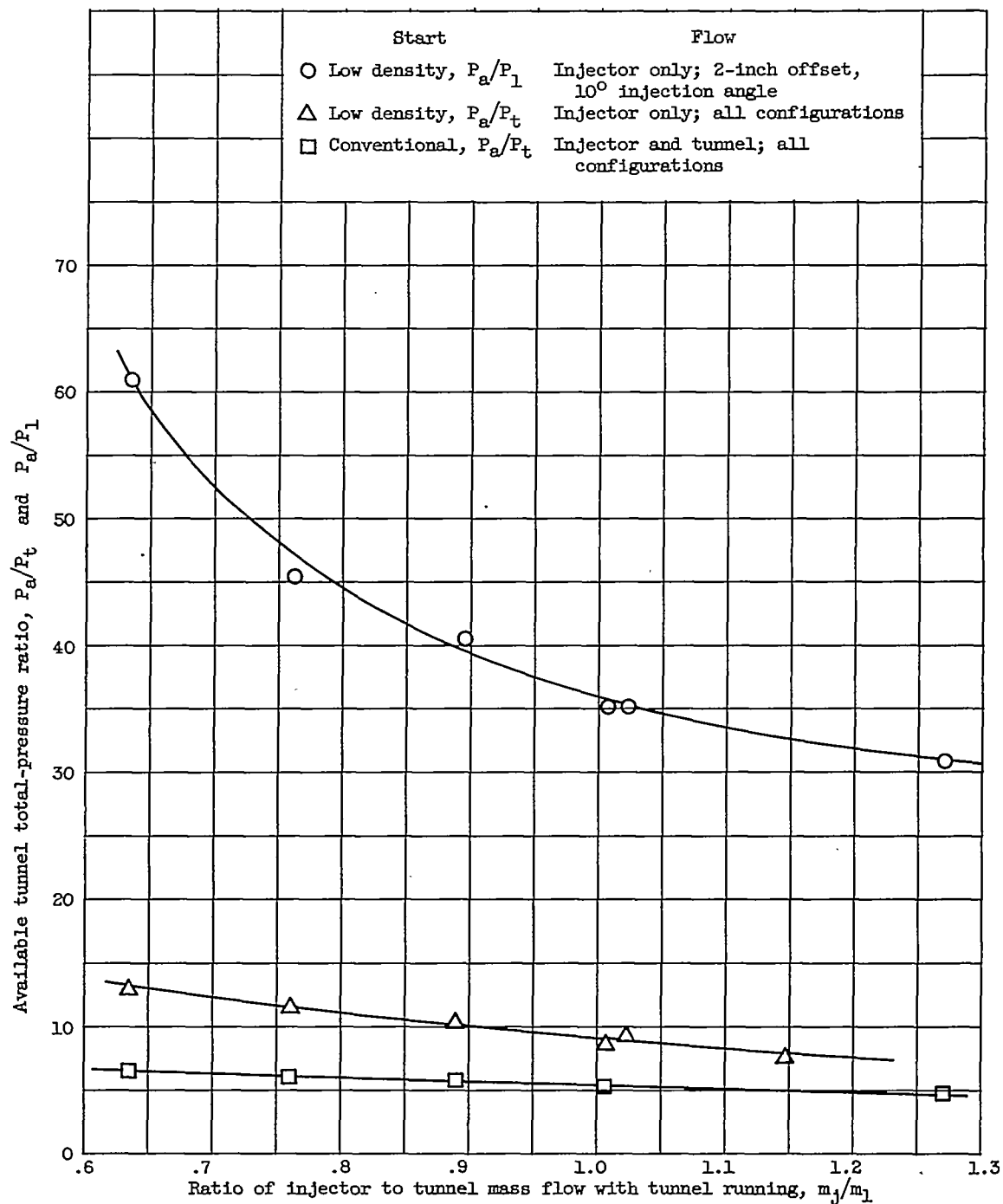


Figure 6. - Comparison of maximum pressure ratios attainable for low density start with those for conventional start. 2- by 2-foot tunnel; Mach number, 3.85.

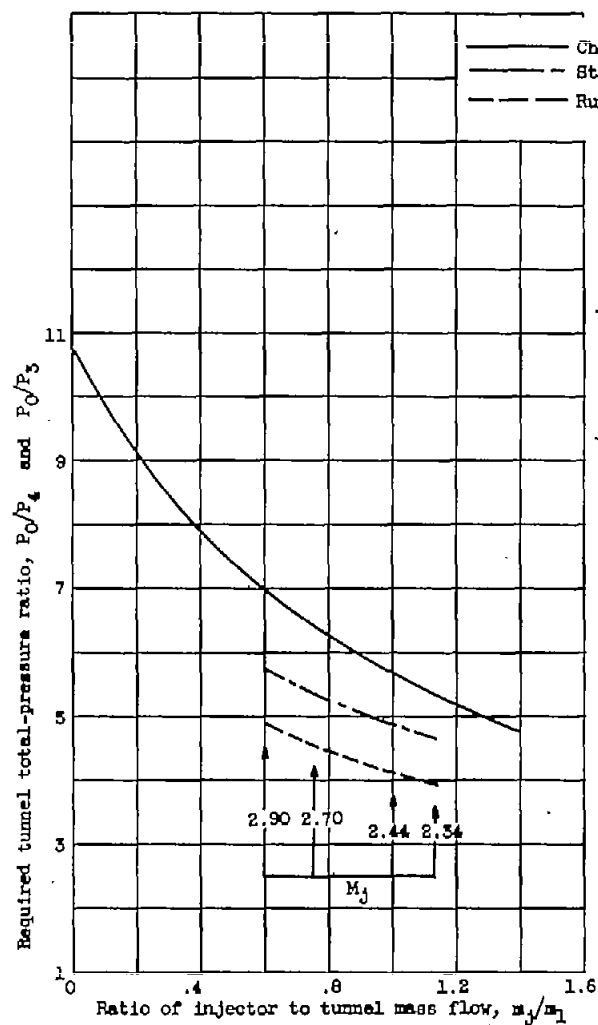
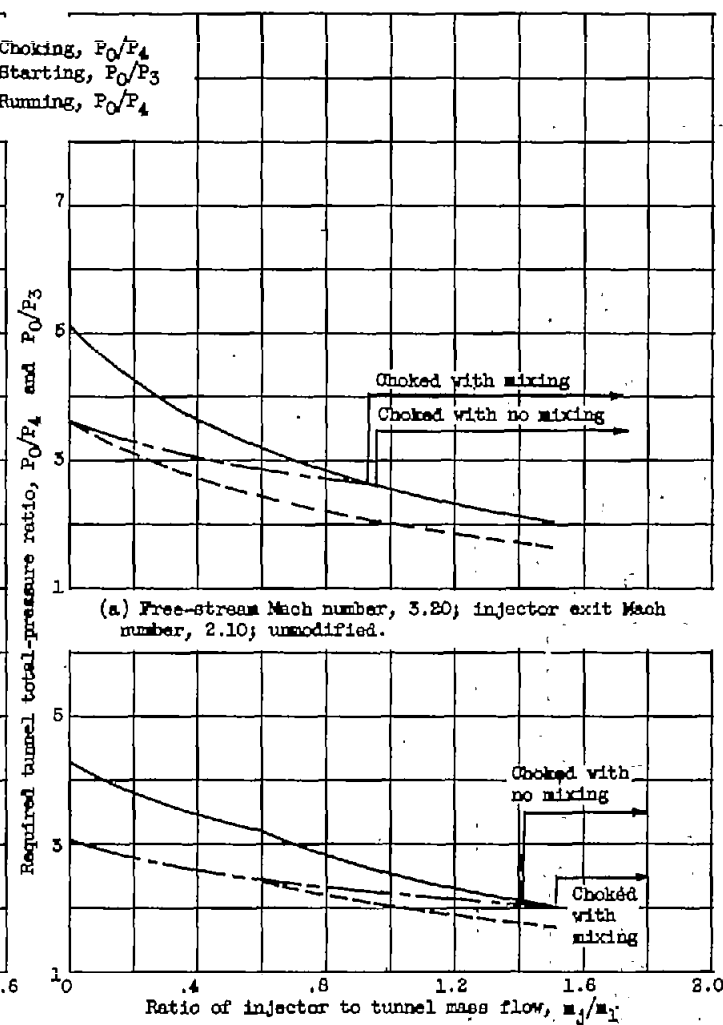


Figure 7. - Theoretical tunnel-injector performance based on 2- by 2-foot tunnel geometry at free-stream Mach number of 4.0



(a) Free-stream Mach number, 3.20; injector exit Mach number, 2.10; unmodified.

(b) Free-stream Mach number, 3.01; injector exit Mach number, 1.80; modified.

Figure 8. - Theoretical tunnel-injector performance based on 18- by 18-inch tunnel geometry.

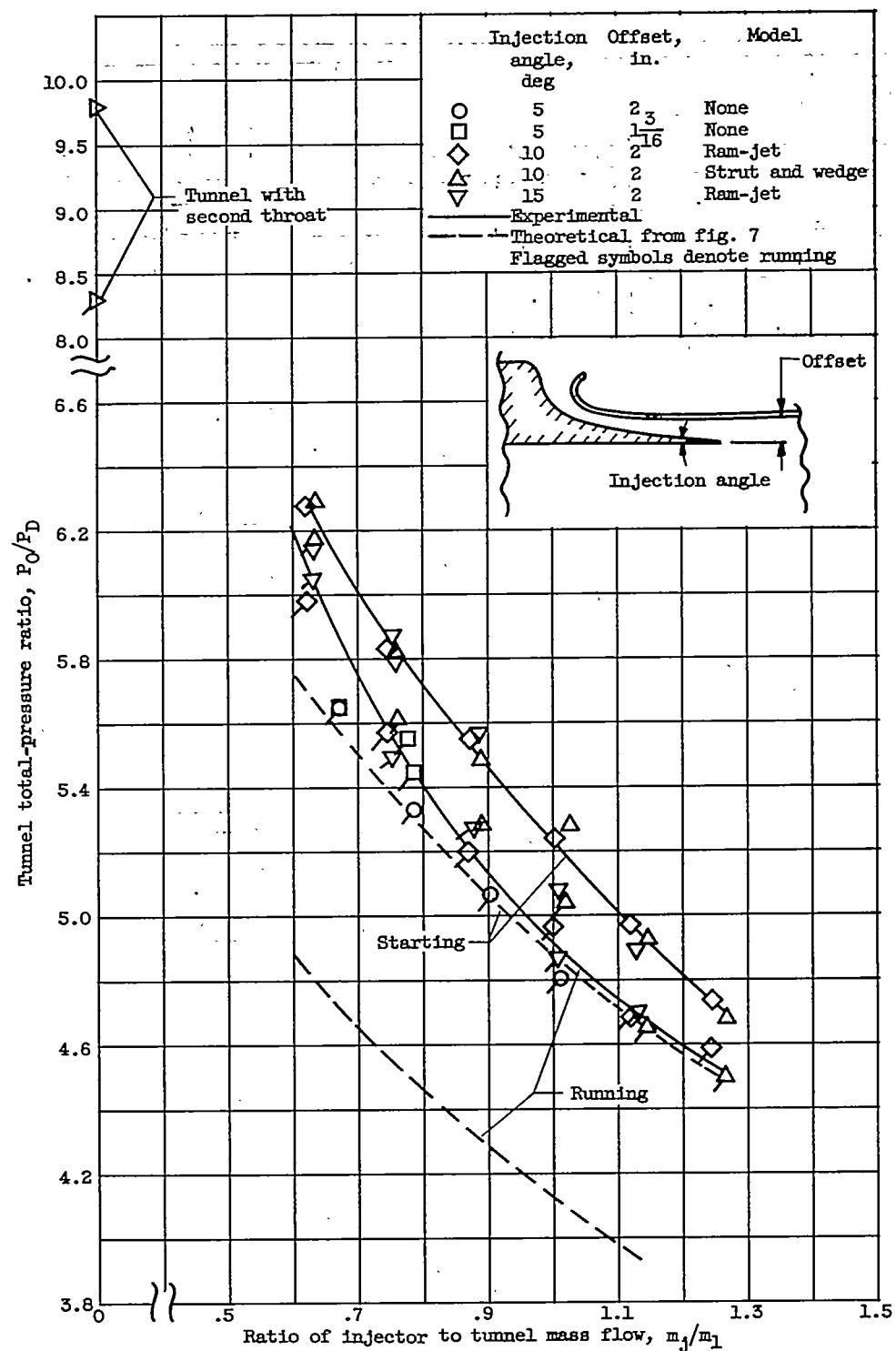


Figure 9. - Experimental 2- by 2-foot tunnel-injector performance.  
Mach number, 3.85. Theoretical curve from figure 7.

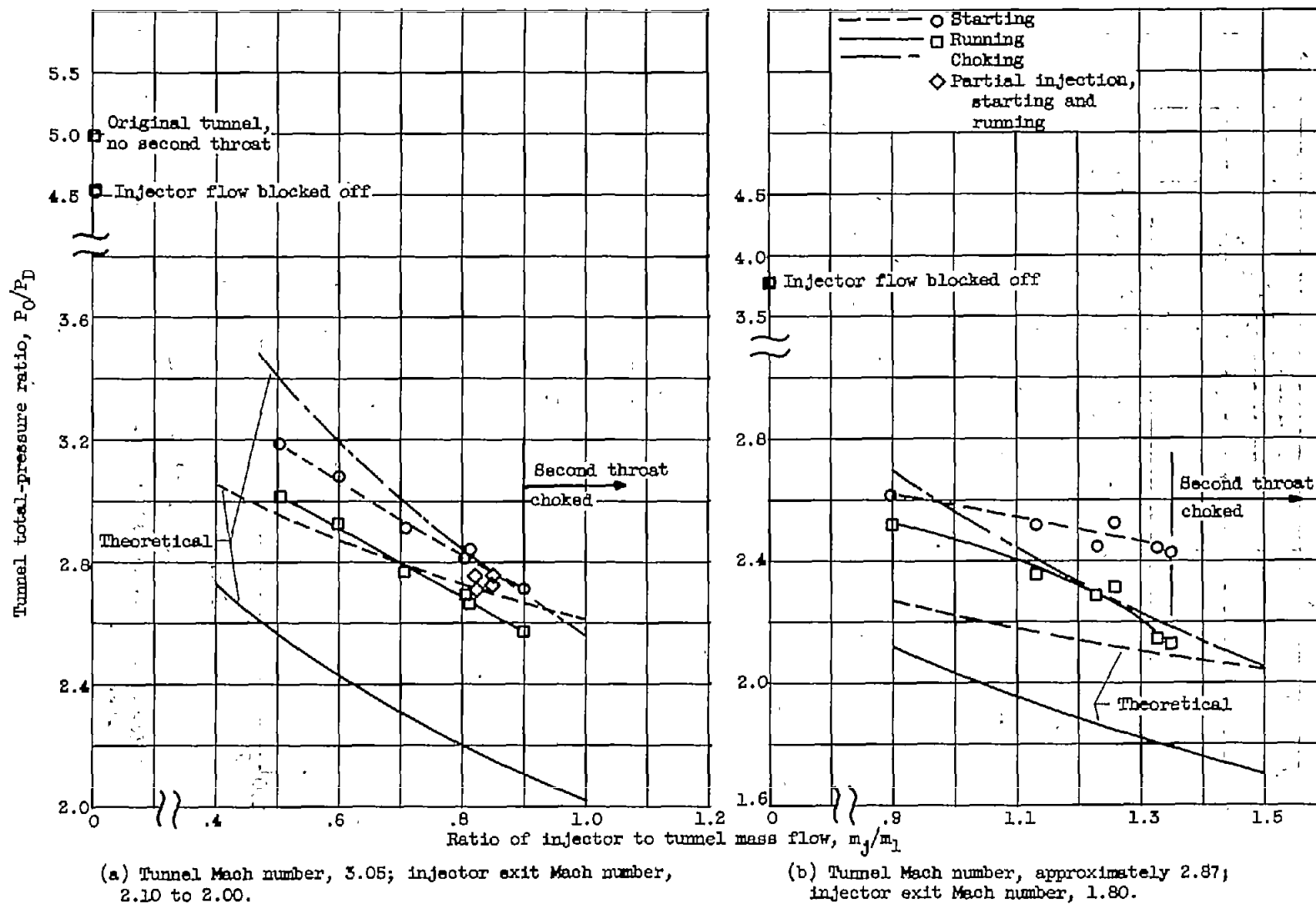
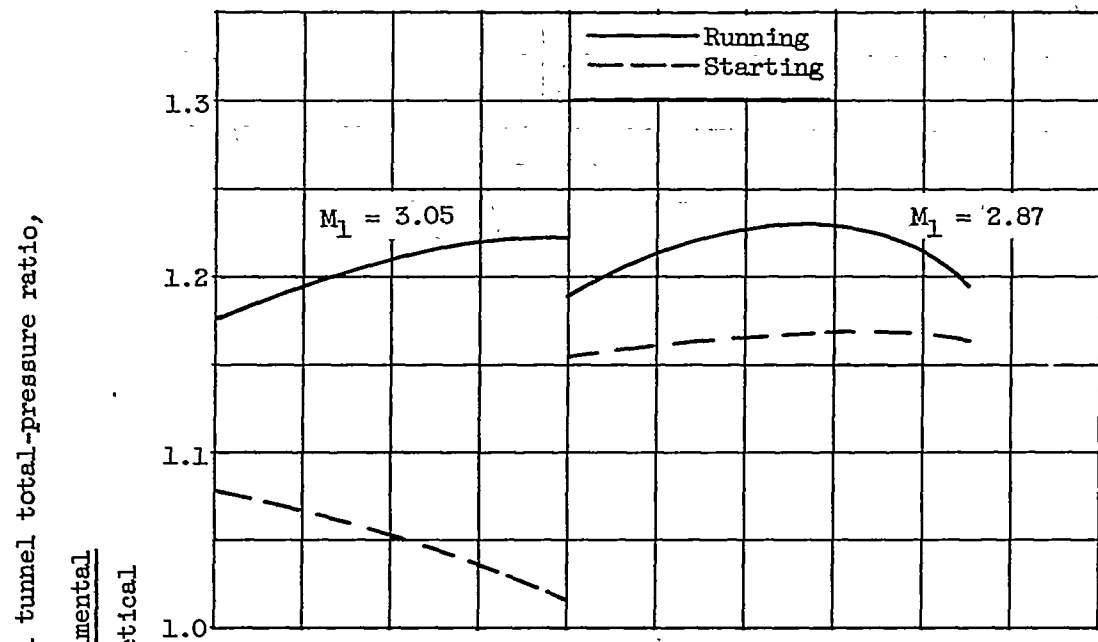
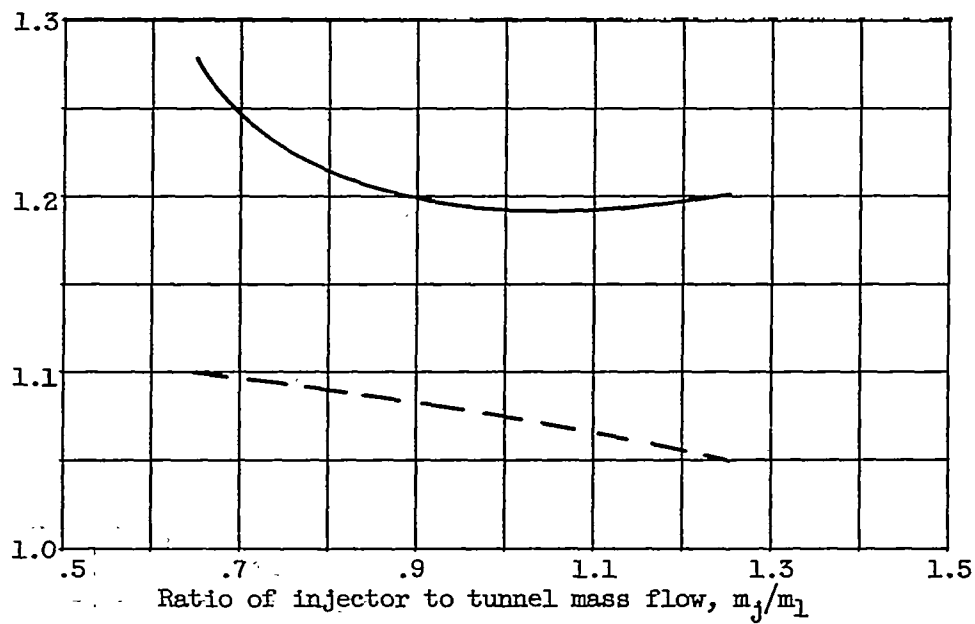


Figure 10. - Experimental 18- by 18-inch tunnel-injector performance. Theoretical curves from figure 8.



(a) Tunnel Mach numbers, 3.05 and 2.87.



(b) Tunnel Mach number, 3.85.

Figure 11. - Comparison of experimental with theoretical tunnel pressure ratio.